Instructors Notes

Title: Mistakes and Errors When Hypothesis Testing

Disciplines/Courses: Contemporary Moral Problems (PHIL 132). This exercise could be used in any introductory philosophy course, or any course that deals with values, like Introduction to Theology.

Degree of Difficulty: Introductory

Abstract: In this module we will look at a decision technique, useful in the face of uncertainty, known as the "hypothesis test." We will learn that while hypothesis testing is not a perfect decision process and that errors can occur; the method also gives us an idea of what kind of error may occur and how often we can anticipate that error.

Resources: No additional resources are needed as the module is self-contained. The computations require only multiplication and division.

Open ended questions: One could do a very similar project using false positives in testing of prostate cancer.

Revision & Continuation: The project can be repeated with a different example (see above). Since this project is one that freshmen can do without knowing much math, moving to a higher degree of difficulty would not be appropriate.

Further Guidelines on Evaluation: Right now, this is a credit/no credit exercise, and it should remain so in introductory classes.

Individual or Group Project: This is a project that will be done in teams of two.

Data: All the data is provided before. This a standalone assignment so no other sources are needed. **[JOHN, THIS IS TRUE UNLESS WE ADD THE VERY SIMPLE SUGGESTED BELOW.]** No background in statistics is necessary. All our computations will only involve division and multiplication.

All of us know or will know a friend or family member who is diagnosed with cancer. A cancer diagnosis is one of the scariest things in the world. These diagnoses are usually the result of being given a test (tests are not perfect). The results are interpreted by an expert (or maybe an expert system on a computer.) As with all hypothesis tests there are four possibilities:

		Gold Standard		
		Cancer	No Cancer	
Test Results	Cancer	tp = true pos	fp = false pos	
	No Cancer	fn = false neg	tn = true neg	

Suppose that for a given type of cancer in a given community there is a 1% prevalence of the disease. Further suppose that if a person has cancer his/her test will be positive 80% of the time.

This is called the sensitivity of the test. And suppose that if a person does not have cancer, they will still get a positive result 10% of the time. This is called false positive.

The numbers given above are called conditional probabilities. For example, "If a person has cancer his/her test will be positive 80% of the time." In that statement the probability of a positive test is CONDITIONED by the event "person has cancer." Hence it is called a CONDITIONAL PROBABILITY.

We can convert these conditional probabilities into counts if we assume we are testing 1000 people. Doing this the numbers look like:

		Gold Standard		
		Cancer	No Cancer	
Test Results	Cancer	tp = 8	fp = 100	108
	No Cancer	fn = 2	tn = 890	892
		10	990	1000

By "Gold Standard" we mean a theoretical way of measuring that has essentially no error. [JOHN SHOULD THE FOLLOWING SENTENCE BE DELETED?] Note: to my regret I never took a class in statistics in high school or university and most of you probably haven't either. To make computations easy we will use the formulas below. This means we can generate very helpful information by just using multiplication and division.

[JOHN, SHOULD WE HAVE A QUESTION WHERE THEY EXPLAIN CONCEPTS OF FALSE POSITIVES AND SO ON? IT WOULD BE EASY FOR THEM TO LOOK UP.]

1. *What is the false positive rate*? To calculate, use the following formula: fp/ (total number with no cancer)*100. Multiplying by 100 makes your answer a percent. 100/990*100 = 10.1%. Using the table above and the formula, calculate the false positivity rate. Did you get the same answer? If not, can you figure out where you went wrong?

2. *What is the sensitivity rate*? Calculate it using the formula tp/(total number with cancer), which is 8/10*100=80%. Using the table above and the formula, calculate the false sensitivity. Did you get the same answer? If not, can you figure out where you went wrong?

3. *If subject A is tested for cancer and returns to his physician a week later and receives the grim news that his test was positive, what is the probability that he actually has cancer? Enter your guess here. ______________ Note to students: you may not be able to calculate an exact answer, this question just asks that you think about the given data and make a thoughtful guess as to the answer*

Using a technique called Bayesian Analysis we can calculate the probability of cancer given the new evidence of a positive cancer test. If you take a statistics course, you will learn this technique but we will continue to use formulas to keep things simple.

The probability that subject A really has cancer, given a positive test result is: (number of true positives) / (total number of positive test) = 8/108 = .0741. By multiplying the decimal by 100 we get a percent for our answer. 7.4%.

4. Using the table above and the formula, calculate the probability of cancer given the evidence of a positive cancer test.

Does this number surprise you? If you were a bit off the number in your guess you are in good company. In a survey of 100 physicians, about 95% estimated the probability their patient had cancer given positive test results to be about 75%. Their estimate was off by a factor of ten. How did you do?

5. *Does it trouble you that in a recent survey given to physicians it was shown that many of the physicians erroneously estimated the probability of cancer given the patient had a positive test result at 75%, when the actual probability is 7.4%? Do you have a suggestion about remedying this situation? If so, what is it?*

6. Notice how useful this information would be if you or someone you loved got a positive test. Most humans are terrible when thinking about large numbers, probabilities, and as a result are very bad at assessing risk. (For example, if you were a parent, it is a lot safer for your child to be playing in a house with a gun than playing in a house with a swimming pool, other things being equal.) Many people during the COVID-19 pandemic got sick, or died, and/or passed it on to vulnerable populations, including their own friends, parents, and grandparents. These deaths were premature, usually preventable, and caused by misinformation and/or someone who didn't take the risk of contracting COVID seriously.

For a general background to what we have been doing read the short article the short article linked below Reading the article provides some context to the issues addressed below. Pay attention to the concept of sensitivity.

https://www.healthnewsreview.org/toolkit/tips-for-understanding-studies/understanding-medical-tests-sensitivity-specificity-and-positive-predictive-value/

The beauty and power of statistics is that it lets you quantify the likelihood of events that otherwise you would be guessing about. Statistical decisions are not always correct, but statistics not only gives us a prediction, in addition statistics gives us a bound on how often it is incorrect. That's a great package deal.

However, we need to careful about how the information conveyed is presented. Percentages are used in many ways to influence our thinking and acting—think advertising and politics—but we often don't know enough to realize that the information as presented is misleading. We don't get (exact) do-overs in life, but if I could return to my university days, I would take statistics. **Of course not! But you could delete it if you want, this is the instructor notes.**

Deliverables and evaluation: You will work in groups of two as homework. One answer per group. The answer needs to include: (1) answers to all questions and a (2) one paragraph summary of this experience. The typed answers will only receive credit if they are turned in before the next class. If they are one minute late they will receive no credit. The next class we will go over your answer.